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Solving Equations – SW 7

7.1 Gleichungen in einer Variablen

Quadratische Gleichung

a) $x^4 - 5a^2x^2 + 6a^4 = 0$ | subst $z = x^2$

$z^2 - 5a^2z + 6a^4 = 0$ | Quad-Gl

$x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$z_{1,2} = \frac{+5a^2 \pm \sqrt{25a^4 - 24a^4}}{2}$

$= \frac{5a^2 \pm \sqrt{a^4}}{2}$

$= \frac{5a^2 \pm a^2}{2}$

$z_1 = 3a^2$

$z_2 = 2a^2$

Quad Gl

Rücksubstitution: 1) $x^2 = 3a^2 \Rightarrow x^2 - 3a^2 = 0 \Rightarrow \underline{\underline{x_{1,2} = +/- a\sqrt{3}}}$

2) $x^2 = 2a^2 \Rightarrow x^2 - 2a^2 = 0 \Rightarrow \underline{\underline{x_{3,4} = +/- a\sqrt{2}}}$

b) $x^4 - 3ax^2 - 4a^2 = 0$ | $z = x^2$

$z^2 - 3az - 4a^2 = 0$ | Quad-Gl

$z_{1,2} = \frac{3a \pm \sqrt{9a^2 + 16a^2}}{2}$

$= \frac{3a \pm \sqrt{25a^2}}{2} = \frac{3a \pm 5a}{2}$

$z_1 = 4a$

$z_2 = -a$

1) : $x^2 = 4a \Rightarrow x = \sqrt{4a} \Rightarrow \underline{\underline{x_{1,2} = +/- 2\sqrt{a}}}$ ✓

2) : $x^2 = -a \Rightarrow x = \sqrt{-a} \Rightarrow \underline{\underline{x_{3,4} = +/- \sqrt{-a}}}$

c) $(x+1)^5 - (x-1)^5 = 23032$

$x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1 - (x^5 - 5x^4 + 10x^3 - 10x^2 + 5x - 1) = 23032$

$10x^4 + 20x^2 + 2 = 23032$

$10x^4 + 20x^2 = 23030$ | $z = x^2$ & : 10

$z^2 + 2z = 2303$

$z^2 + 2z - 2303 = 0$ | Quad Gl $-1 \pm \sqrt{1 + 2303}$

$z_1 = 47$

$z_2 = -49$

1): $x^2 = 47 \Rightarrow \underline{\underline{x_{1,2} = +/- \sqrt{47}}}$

2): $x^2 = -49 \Rightarrow$ nicht möglich

Hornerschema

a) $4x^3 - 8x^2 - 11x + 15 \quad x_0 = 5 \,/\, x_0 = -4$

	4	-8	-11	15
$x_0 = 5$	0	20	60	245
	4	12	49	260

$(x-5)(4x^2+12x+49)+260$

$\underline{\underline{p(5) = 260}}$

5 als Lösung abspalten
(x − 5) · "Term" + "Lösung"

	4	-8	-11	15
$x_0 = -4$	0	-16	96	-340
	4	-24	85	-325

$\underline{\underline{p(-4) = -325}}$

b) $-2x^6 + 13x^4 + 25x^2 - 17 \quad x_0 = 3 \,/\, x_0 = \sqrt{5}$

	x^6	x^5	x^4	x^3	x^2	x^1	x^0
	-2	0	13	0	25	0	-17
$x_0 = 3$	0	-6	-18	-15	-45	-60	-180
	-2	-6	-5	-15	-20	-60	-197

$\underline{\underline{p(3) = -197}}$

	-2	0	13	0	25	0	-17
$x_0 = \sqrt{5}$	0	-4,47	-10	6,71	15	89,44	200
	-2	-4,47	3	6,71	40	89,44	183

$\underline{\underline{p(\sqrt{5}) = 183}}$

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SS 7

Felix Rehmer

Auf algebraische zurückführbare Gleichungen

a) $\frac{15x^2 - 26x + 8}{5x-2} = 3x - 5$ $\quad | \cdot 5x-2$

$5x - 2 = 0$
$5x = 2$
$x = 2/5$

$\underline{R \setminus \{2/5\}}$

$\cancel{15x^2} - 26x + 8 = (5x-2)(3x-5)$
$\quad \| \qquad = \cancel{15x^2} - 25x - 6x + 10$
$5x = 2$
$x = 2/5$

$\underline{\underline{\mathbb{L} = \{\}}}$ // $x = 4/5$ darf nicht sein, sonst div 0!

b) $\frac{(x-\frac{3}{2})^2}{x} = \left(\frac{x-1}{x}\right)(x-2) + \frac{1}{4} \cdot \frac{1}{x}$ $\quad | \cdot 4x \quad R \setminus \{0\}$ ✓

$4(x-\frac{3}{2})^2 = 4(x-1)(x-2) + 1$

$4(x^2 - 3x + 9/4) = (4x-4)(x-2) + 1$

$\cancel{4x^2} \cancel{-12x} + 9 = \cancel{4x^2} \cancel{-12x} + 8 + 1$

$9 = 9 \quad \Rightarrow 0 = 0$

$\underline{\underline{L = R \setminus \{0\}}}$

c) $e^x = 1 + \frac{1}{e^x}$ $\quad | \cdot e^x$

$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$(e^x)^2 = e^x + 1$

$(e^x)^2 - e^x - 1 = 0$ $\quad | \; z = e^x$

$(z)^2 - z - 1 = 0$ $\quad |$ Quad-GL

$z_{1,2} = \frac{1 \pm \sqrt{5}}{2}$

$x_1 = \ln\left(\frac{1+\sqrt{5}}{2}\right)$

$\underline{\underline{L = \{\ln(\frac{1+\sqrt{5}}{2})\}}}$

$e^x = z$
$x = \ln(z)$

x_2 ist nicht möglich, da <0!

d) $(16^x - 1)^3 - 49(16^x - 1) = 0$

$\underline{L_1 \; x = 0}$ $\quad 10 - 0 = 0$ ✓

$(16^x - 1)^3 = 49(16^x - 1)$ $\quad | : 16^x - 1$ Fall $16^x - 1 \neq 0 \Rightarrow x \neq 0$

$(16^x - 1)^2 = 49$ $\quad | \; z = 16^x - 1$

$z^2 - 49 = 0$ $\quad |$ Quad-GL

$z_{1,2} = +/- 7$
$z = 16^x - 1$

z_1: $16^x - 1 = 7$
$16^x = 8$
$x = \log_{16}(8)$
$\underline{x_1 = 3/4}$

z_2: $16^x - 1 = -7$
$16^x = -6 \quad <0!$
nicht möglich

$\underline{\underline{L = \{0, 3/4\}}}$ ✓

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SS7

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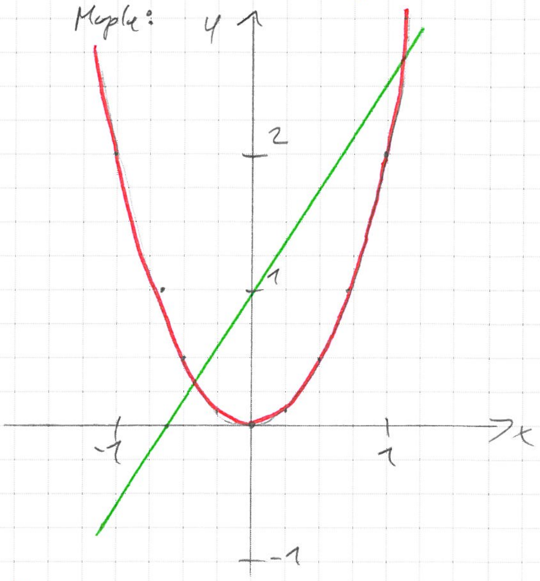
Numerisch zu lösende Gleichung

$$2x^2 = 3x/2 + 1$$

Alle drei Verfahren finden (Maple) den Schnittpunkt. Jedoch in der Regel nur einen. d.h. der Range muss explizit angegeben werden!

Newton Verfahren ist am "schnellsten".

$x_1 = 1,1754$ $x_2 = -0,4254$



7.2 Ungleichungen in einer Variablen

$$\frac{x^2 + x - 12}{x^3 - 4x} \geq 0$$

$x^3 - 4x = 0$

$x(x^2 - 4) = 0$ ⇒ $x \neq 0$

$x^2 - 4 = 0$ ⇒ $x \neq +/-2$

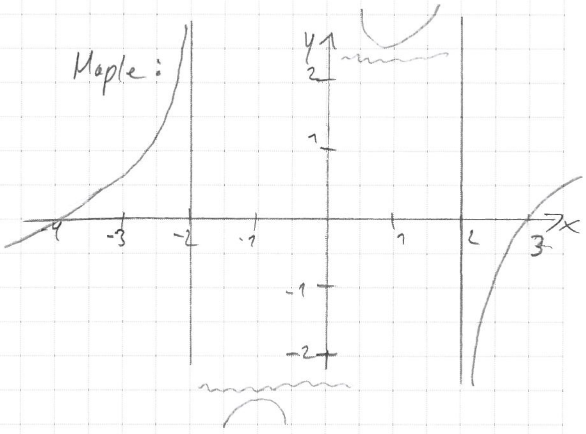
$D = \mathbb{R} \setminus \{-2, 0, 2\}$

$x^2 + x - 12 = 0$ | Quad-Gl

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x_{1,2} = \frac{-1 \pm \sqrt{1+48}}{2} = \begin{matrix} x_1 = 3 \\ x_2 = -4 \end{matrix}$$

$L = [-4, -2[\vee]0, 2[\vee [3, \infty[$



7.3 System von linearen Gleichungen

$$\begin{aligned} kx + y &= 1 \\ x + ky &= 1 \\ kz &= 1 \end{aligned}$$

$k \in R$

Ges: k
- keine
- genau eine
- unendlich viele Lösungen?

$$\begin{array}{ccc|cl} \multicolumn{3}{c|}{\overbrace{\qquad\qquad\qquad}^{\text{Koeffiz.}}} & \overbrace{\;}^{\text{RHS}} & \\ k & 1 & 0 & 1 & \cdot\left(-\frac{1}{k}\right) \\ 1 & k & 0 & 1 & \leftarrow \\ 0 & 0 & k & 1 & \end{array}$$

$$\left[\begin{array}{ccc|c} k & 1 & 0 & 1 \\ 0 & k-\frac{1}{k} & 0 & 1-\frac{1}{k} \\ 0 & 0 & k & 1 \end{array}\right]$$

z: $\Rightarrow z \cdot k = 1 \Rightarrow \underline{z = 1/k}$ (falls $k \neq 0$) (und $1/k \neq 0) \Rightarrow 1$

y: $(k - 1/k) \cdot y = 1 - 1/k$

$$\left(\frac{k^2-1}{k}\right) \cdot y = \frac{k-1}{k}$$

$$\underline{y} = \frac{\cancel{(k-1)} \cdot \cancel{k}}{\cancel{k} \cdot \cancel{(k^2-1)}} = \underline{\frac{1}{k+1}} \quad \text{(falls } k \neq -1)$$

$$\cancel{(k-1)}\,(k+1)$$

x: $k \cdot x + \frac{1}{k+1} = 1$

$$kx = \frac{k+1-1}{k+1} = \frac{k}{k+1}$$

$$\underline{x = \frac{1}{k+1}} \quad \text{(falls } k \neq -1)$$

- $k = 0$ oder -1 ⇒ keine Lösunge ✓
- $k = R \setminus \{-1, 0, 1\}$ ⇒ eine Losung $\left(\frac{1}{k+1}, \frac{1}{k+1}, \frac{1}{k}\right)$ ✓
- $k = 1$ ⇒ unendlich viele Lösungen ✓ da $z = 1 \Rightarrow t \cdot y = 0$

Matrixgleichung

$$A^2 \cdot x^T \cdot (A^2)^{-1} = B \qquad | \cdot (A^2)^{-1}$$

$$E \cdot X^T \cdot (A^2)^{-1} = (A^2)^{-1} \cdot B \qquad | \cdot A^2$$

$$E \cdot X^T \cdot E = (A^2)^{-1} \cdot B \cdot A^2 \qquad | \text{ Vereinfachen}$$

$$X^T = (A^2)^{-1} \cdot B \cdot A^2 \qquad | \; X^T \Rightarrow X \quad \text{!Reihenfolge ändert!}$$

$$X = (A^2)^T \cdot (B)^T \cdot ((A^2)^{-1})^T \qquad | \text{ Vereinfachen}$$

$$X = (A^2)^T \cdot B^T \cdot ((A^2)^T)^{-1}$$

Maple: X:= Transpose (A.A). Transpose(B). MatrixInverse (Transpose (A.A))

$$X: \begin{bmatrix} 1 & 0 & a^2/c^2 \\ 0 & 1 & 0 \\ 0 & c^2/b^2 & 1 \end{bmatrix}$$

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SJ 7

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Textaufgabe und Gleichungssystem

$$x + y + z = 18$$

$$100x + 10y + z = 100y + 10x + z - 180$$

$$100x + 10y + z = 100x + 10z + y - 18$$

Gleichungssystem : Maple : $x = 4$, $y = 6$, $z = 8$

Die gesuchte Zahl lautet 468.

Gleichungssystem und Gauss-Algorithmus

$$\begin{aligned} x + ay + a^2z &= 0 \\ x + 2ay + 2a^2z &= 0 \\ x + by + abz &= 0 \end{aligned}$$

$$\left[\begin{array}{cccc} 1 & a & a^2 & 0 \\ 1 & 2a & 2a^2 & 0 \\ 1 & b & ab & 0 \end{array}\right] \quad \cdot(-1)\ +, \quad \cdot(-1)\ +$$

$$\left[\begin{array}{cccc} 1 & a & a^2 & 0 \\ 0 & a & a^2 \text{①} & 0 \\ 0 & b-a & \underset{a(b-a)}{ab-a^2} & 0 \end{array}\right] \quad \Leftarrow \text{Fall: } a=0 \qquad \cdot\left(-\frac{b-a}{a}\right)\ + \quad \text{Fall } a \neq 0$$

$$\text{①}\ \frac{a^2(b-a)}{a} = -a(b-a)$$

Fall: $a = 0$,

$$\left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & b & 0 & 0 \end{array}\right] \begin{array}{l} \rightarrow \underline{x = 0} \quad ③ \\ \dashrightarrow 0 \cdot z = 0 \Rightarrow \underline{z = u \in R} \quad ② \\ b=0 \rightarrow b \cdot y = 0 \Rightarrow \underline{y = t \in R} \quad ① \Leftarrow b = 0\,! \Rightarrow L_1(a, b=0) \\ b \neq 0 \rightarrow b \cdot y + 0 \cdot z = 0 \Rightarrow y = 0,\ z = t \in R \Leftarrow b \neq 0\,! \Rightarrow L_2(a=0, b\neq 0) \end{array}$$

Fall: $a \neq 0$

$$\left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & a & a^2 & 0 \\ 0 & 0 & \underbrace{\begin{matrix} a(b-a) \\ -a(b-a) \end{matrix}}_{0} & 0 \end{array}\right] \begin{array}{l} \rightarrow \underline{x = 0} \\ \rightarrow a \cdot y + a^2 \cdot t = 0 \Rightarrow y + at = 0 \Rightarrow \underline{y = -at} \\ \rightarrow 0 \cdot z = 0 \Rightarrow \underline{z = t \in R} \Rightarrow L_3(a \neq 0) \end{array}$$

$$L_1(a=0, b=0) = \{(x,y,z) \mid x = 0,\ y = t,\ z = u\ ;\ t, u \in \mathbb{R}\}$$

$$L_2(a=0, b \in \mathbb{R}\setminus\{0\}) = \{(x,y,z) \mid x = 0,\ y = 0,\ z = t\ ;\ t \in \mathbb{R}\}$$

$$L_3(a \in \mathbb{R}\setminus\{0\}, b \in \mathbb{R}) = \{(x,y,z) \mid x = 0,\ y = -at,\ z = t\ ;\ t \in \mathbb{R}\}$$

$$L = L_1 \vee L_2 \vee L_3$$