Integralrechnung – Übung I

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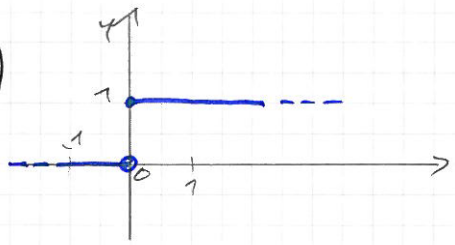
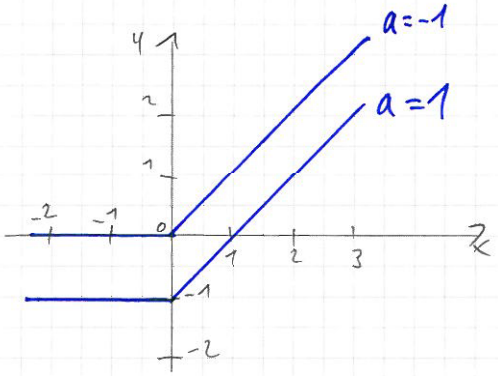
1)

$f(x)$	$F(x)$
0	$c+5$, $c \in \mathbb{R}$
x^{α} , $\alpha \neq -1$	$\frac{x^{\alpha+1}}{\alpha+1} + k$, $k \in \mathbb{R}$
e^x	e^{x+c} , $c \in \mathbb{R}$
$2 \cdot \sin(2x)$	$-\cos(2x)$
$\frac{3}{\sqrt{1-9x^2}}$	$\arcsin(3x) - 1$
3	$3x - c$, $c \in \mathbb{R}$
$\frac{1}{3x}$	$\ln\lvert 3x \rvert - 5$
e^{ax}	$\frac{e^{ax}}{a+c}$, $c \in \mathbb{R}$
$\cos(x/2)$	$2 \cdot \sin(x/2) + 1$
$\frac{1}{3\sqrt{1-(x/3)^2}}$	$\arccos(x/3) + c$, $c \in \mathbb{R}$

2)



$$f(x) \begin{cases} 0, & -\infty < x < 0 \\ 1, & 0 \leq x < \infty \end{cases}$$



$a=-1$: $F(x) \begin{cases} 0 & , \quad -\infty < x < 0 \\ x & , \quad 0 \leq x < \infty \end{cases}$

$a=1$ $F(x) \begin{cases} -1 & , \quad -\infty < x < 0 \\ x-1 & , \quad 0 \leq x < \infty \end{cases}$ $x-1 \neq 1$

4) $f(x) = x^2$
$F(x) = ?$

$F(x) = \frac{x^3}{3}$

$A_{F(x)} = \int_0^1 f(x)\,dx = F(x)\Big|_0^1$
$= F(1) - F(0) = \frac{1^3}{3} - \frac{0^3}{3} = \frac{1}{3} - 0 = \frac{1}{3}$ ✓

5) $\int \frac{f'(x)}{f(x)}\,dx = ?$

FS. S71: $\int_a^b \frac{f'(x)}{f(x)}\,dx = \left[\ln|f(x)|\right]_a^b$ ✓

6) $\int \cot(x)\,dx = ?$

$\int \cot(x)\,dx \overset{?}{=} \int \frac{\cos(x)}{\sin(x)}\,dx$
$= \frac{[\sin(x)]'}{\sin(x)} \Rightarrow$ Aufg 5 $\Rightarrow \ln(|\sin(x)|) + c$ ✓

7) $f(x) = \frac{1}{1+x^2} \Rightarrow F(x) = ?$

$[F(x)]' = f(x) = \frac{1}{1+x^2}$

FS. S.65 $\int \frac{1}{1+x^2}\,dx = \arctan(x) = F(x)$

$\frac{d}{dx}(\arctan(x)) = \frac{1}{1+x^2}$

$\int_{-1}^1 f(x)\,dx = \int_{-1}^1 \frac{1}{1+x^2} = F(x)\Big|_{-1}^1$
$= F(1) - F(-1)$
$= \arctan(1) - \arctan(-1)$
$= \frac{1}{2}\pi$